Benchmark Problems for Beam Models in Flexible Multibody Dynamics

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Outline of the presentation

1. Introduction

2. Proposed benchmark problems
   - The Princeton beam experiment
   - Pure bending of a uniform beam
   - Crooked four bar mechanism
   - Lateral buckling of an I-beam
   - Stability of a rotating shaft

Introduction: multibody formulations

- As the need to model flexibility arose in multibody dynamics, the floating frame of reference formulation was developed.
- This approach can yield inaccurate results when elastic deformations becomes large.
- New formulations are developed
  1. Geometrically exact beam formulations (Simo, 1985)
  2. Three-dimensional beam formulations (Borri, 1983, 1992)
  3. Absolute nodal coordinate formulation (Shabana, 1997)
- New solution strategies are also proposed
  1. Intrinsic beam formulations (Hegemier, 1977, Hodges, 1990)
  2. A DAE approach to flexible multibody dynamics (Betsch *et al.*)
  3. Lie group time integrators in multibody dynamics (Brüls *et al.*)

Introduction: beam theories

- Numerous beam theories have been developed by assuming specific deformation of their cross-section.
  1. In Euler-Bernoulli beam theory, cross-sections are assumed to remain plane and normal to the deformed axis of the beam.
  2. In Timoshenko beam theory, cross-sections are assumed to remain plane but not necessarily normal to the deformed axis of the beam axis.
- For long beams with closed cross-sections made of homogeneous, isotropic materials, these assumptions
  1. are in close agreement with experimental observations and
  2. predictions based on these theories yield sufficient accuracy for many engineering applications.
- Therefore, these assumptions underpin the developments found in most beam theories used for multibody dynamic simulations.

Introduction: need for assessment

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- Clearly, rapid developments are taking place for both formulations and solution procedures of beam problems
- The same remarks could be made concerning plate and shell problems
- Systematic comparisons are needed to assess
  1. The accuracy of the various formulations
  2. The computational performance of the various solution strategies
- Without these systematic comparisons no progress is possible

Introduction: benchmark problems

- In many areas, benchmark problems play an important role
  1. In the early phases of development of the finite element method, NASA set up a special tool for the validation of new finite elements
  2. Prof Schiehlen presented a review of the benchmark problems used in multi rigid body dynamics
  3. In flexible multibody dynamics, the many recent developments should be compared in a systematic manner,
- Good benchmark problem are difficult to choose!
  1. Must be defined clearly
  2. Must address difficulties one at a time
  3. Must be made available to all
- New formulations should be presented only if they simulate these benchmark problems successfully

Introduction: benchmark problems

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1. Benchmark problems should involve
   - Experimental data sets: validate the formulation
   - Analytical data sets: validate the accuracy of the solution process
   - Numerical data sets: experimental or analytical solutions are not available
2. Desirable features of benchmark problems
   - Data sets for various physical feature of the problem
     - static behavior (Bending, torsion, coupling),
     - dynamics (inertia, gyroscopic),
     - material (isotropic, composites),
     - stability (elastic stability, dynamic stability, LCO).
   - Provide very detailed information (displacements, velocities, 3D stresses and strains, . . .)
   - Data sets of increasing difficulty to identify of problem areas
   - Data sets for each structural element: beam, plate, shells, 3D, . . .

Introduction: use of benchmark problems

Benchmark problems are used to

1. Validate the formulation (use experimental data sets)
2. Assess the accuracy of the solution (use analytical data sets)
3. Establish adequacy of the framework (use numerical data sets)
4. Make you feel good (Find the “same answer as many others”)
5. Establish computation efficiency (Difficult to do!)

Introduction: objective of the paper

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1. This paper presents a series of benchmark problems for beam elements used in flexible multibody dynamics
   1. The Princeton beam experiment: experimental data set
   2. The pure bending of a uniform beam: analytical data set
   3. The crooked four bar mechanism: challenging data set
   4. The lateral buckling of an I-beam: challenging data set
   5. The stability of a rotating shaft: challenging data set
2. For each benchmark problem
   1. Description of the problem
   2. Identification of the challenge
   3. Preliminary results

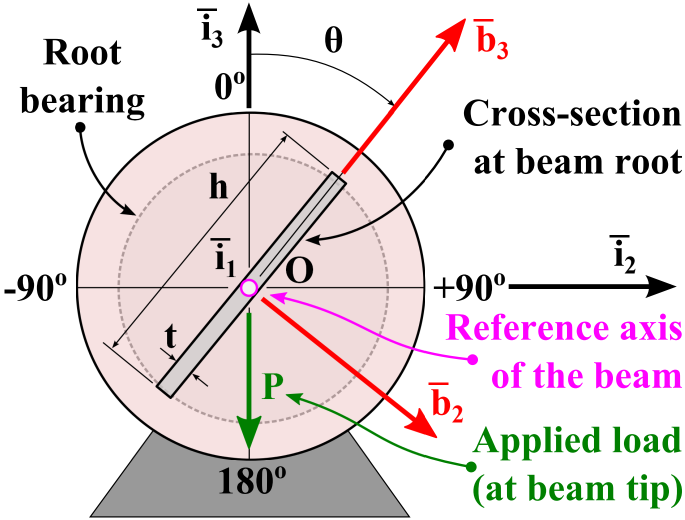
The Princeton beam experiment

1. The Princeton beam experiment[1] is a study of the static large displacement and rotation behavior of a simple cantilevered beam under a gravity tip load[2]
2. A straight aluminum (T 7075) beam
   - length $L = 0.508$ m
   - a rectangular cross-section of thickness $t = 3.175$ mm and height $h = 12.7$ mm
   - cantilevered at its root and
   - subjected to a static concentrated load P at its tip.

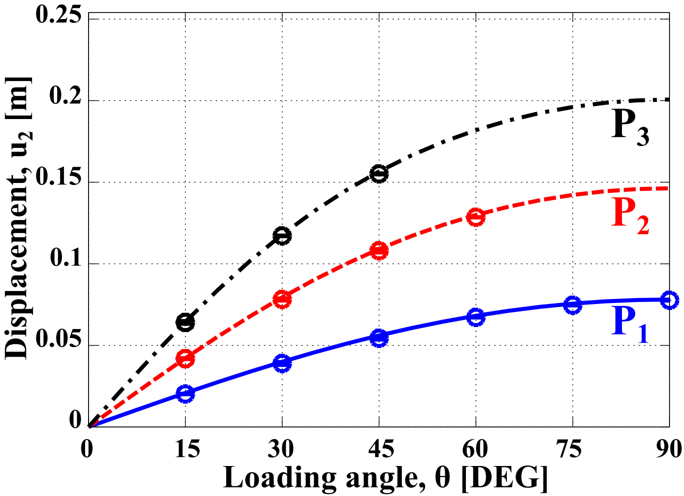
[1] E. H. Dowell and J. J. Traybar. An experimental study of the nonlinear stiffness of a rotor blade undergoing flap, lag, and twist deformations. Aerospace and Mechanical Science Report 1257, Princeton University, 1975.

[2] E.H. Dowell, J. Traybar, and D.H. Hodges. An experimental-theoretical correlation study of non-linear bending and torsion deformations of a cantilever beam. *Journal of Sound and Vibration*, 50(4):533- 544, 1977.

The Princeton beam experiment

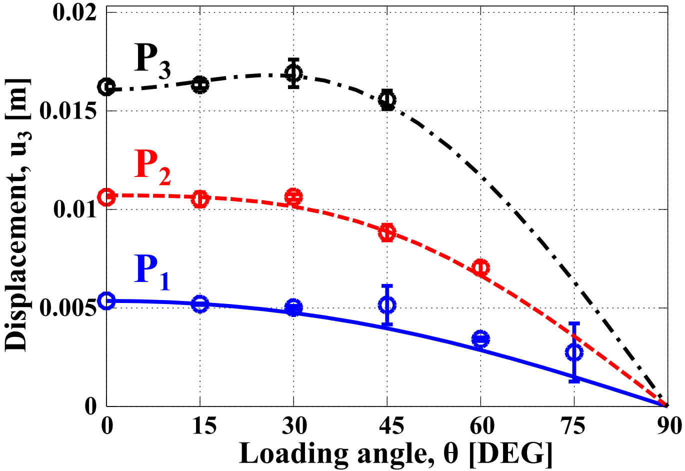


Experimental results in soft direction



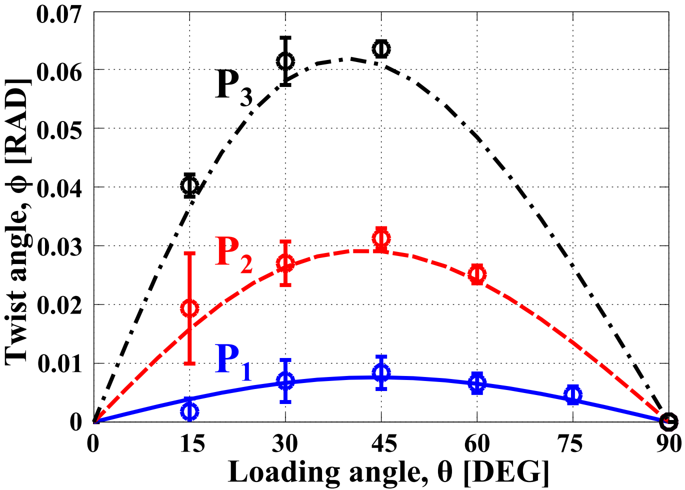
Measured transverse displacements in the soft direction for P_1, P_2, and P_3 = 1, 2, and 3 lbs, respectively.

Experimental results in stiff direction



Measured transverse displacements in the stiff direction for P_1, P_2, and P_3 = 1, 2, and 3 lbs, respectively.

Experimental results in torsion



Measured transverse twist for P_1, P_2, and P_3 = 1, 2, and 3 lbs, respectively.

Pure bending of a uniform beam

1. Under a tip bending moment, a uniform beam deforms into a circular arc.
2. Note that this solutions is exact
   - for the geometrically exact beam formulation only,
   - Assumes homogeneous, isotropic material,
   - Neglects all end effects

Crooked four bar mechanism

1. Deals with the dynamic response of a four bar mechanism with one “off axis” revolute joint
2. This nearly static problem is challenging because of the coupled bending-torsion nature of the deformation
3. Data will be presented in web format

Definition of physical properties

1. The definition of physical properties calls special attention
2. Definition of geometric properties is rather obvious
3. Definition of material properties is far more difficult
   - Some formulations (GEBF), use sectional properties (bending stiffness, torsional stiffness, ...) as basic inputs
   - Some formulations (3DBF, ANCF), use material properties (Young's modulus, Poisson's ratio, ...) as basic inputs
4. These differences might complicate the interpretation of the results
5. The type of properties used should be defined clearly when reporting results

Lateral buckling of an I-beam

1. Deals with the constrained lateral buckling of a beam under tip loading
2. This dynamic problem is challenging because of the very violent nature of this elastic stability problem
3. Data will be presented in web format

Stability of a rotating shaft

1. Deals with the dynamic of a cantilevered beam rotating at high speed about its own axis
2. This dynamic problem is challenging because of the gyroscopic nature of the instability
3. When material dissipation is present, a Limit Cycle Oscillation is observed
4. Details of this example are under preparation

Conclusions

- A number of benchmark problems for beam models used in flexible multibody dynamics has been proposed.
- Proposed benchmark problems include
    1. Experimental data sets,
    2. Analytical data sets, and
    3. Numerical data sets
- Proposed benchmark problems include
    1. static deformation,
    2. coupled bending-torsion in 3D,
    3. gyroscopic dynamics,
    4. elastic stability

Conclusions

- All problems will be
  1. fully documented,
  2. made available to the community via web page
- This is a community effort
  1. Tabulated numerical data will be available for all to compare
  2. Additional problems should complement those proposed here
- A paper presenting a four way comparison (four independent codes) is under preparation