A Library of Computational Benchmark Problems

for the Multibody Dynamics Community

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01-04 July 2013

Outline

1. Introduction
2. Systematic classification of multibody benchmark problems
3. Evaluation of benchmark problems
4. The Library of computational benchmark problems
5. Summary and future work

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Motivation

Why a Library of Computational Multibody Benchmark Problems?

- A comprehensive reference for current and future generations of researchers
- A tool for sharing our computational experience:
  - Modeling schemes
  - Formulation methods
  - Numerical procedures
  - Software implementation
- A resource, facilitating collaboration and cross-pollination of ideas
- A collaborative means to
  - Introduce new benchmark problems in our ever-growing field
  - Classify multibody systems and simulation procedures
  - Discuss systematic treatment of various multibody systems

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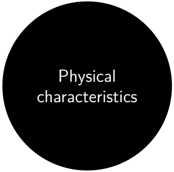
Classification schemes

✷ Physical characteristics and applications



Classification schemes

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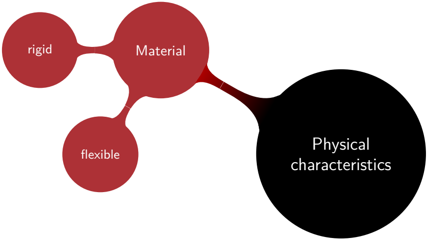

Classification schemes

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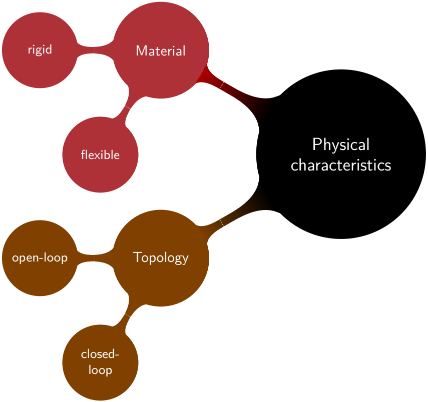

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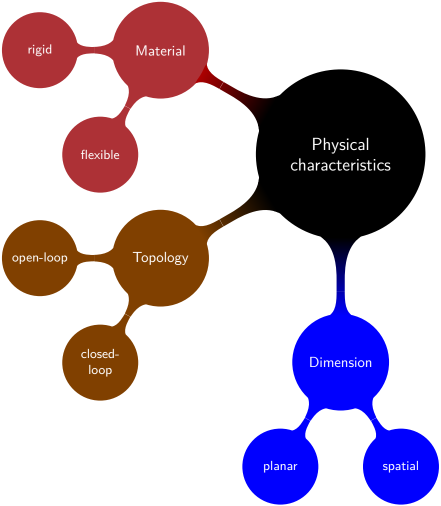

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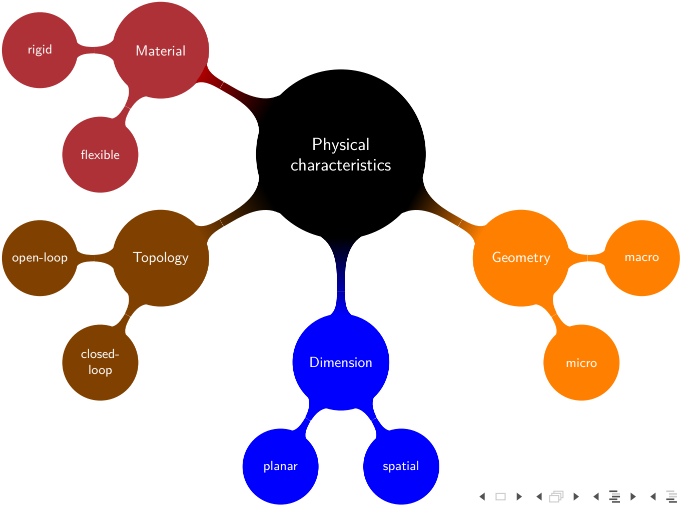

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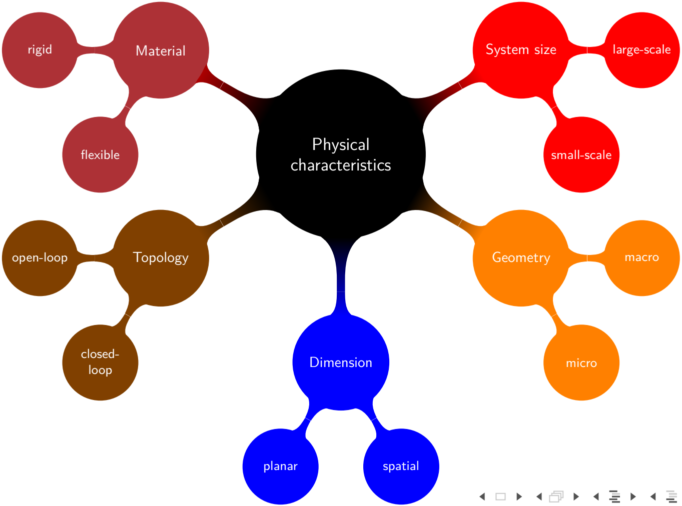

Classification schemes

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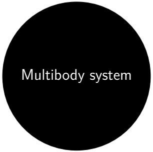

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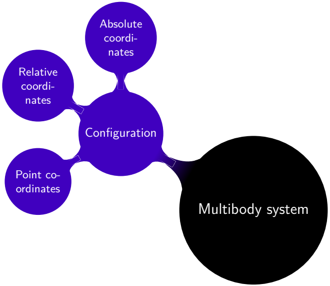

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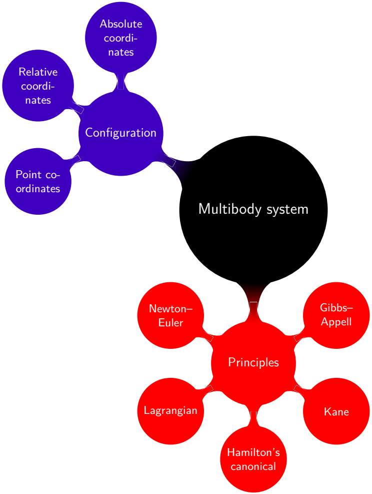

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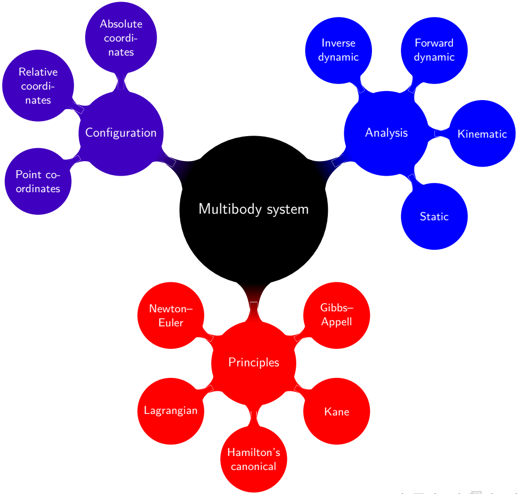

Classification schemes

✷ Simulation methods and computational issues:

Formulating the dynamic equations

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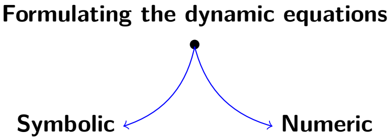
Classification schemes

Simulation methods and computational issues:



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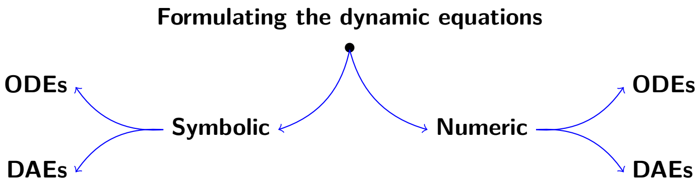

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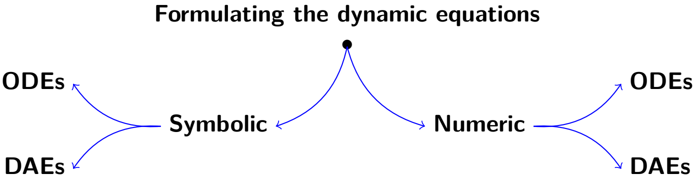

Formulation schemes

Recursive methods
Body-coordinate formalism
Penalty formulations
Velocity transformations
Baumgarte stabilization
Linear graph theory

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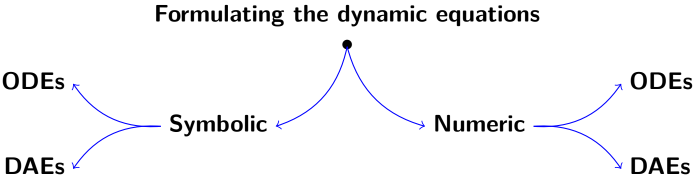

Formulation schemes

Recursive methods
Body-coordinate formalism
Penalty formulations
Velocity transformations
Baumgarte stabilization
Linear graph theory

Integration methods

Implicit/Explicit algorithms
Single- and multi- step methods
Forward/backward methods
Recursive methods

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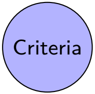

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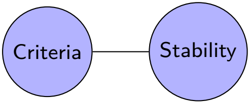

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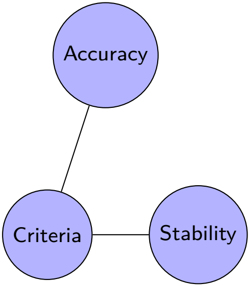

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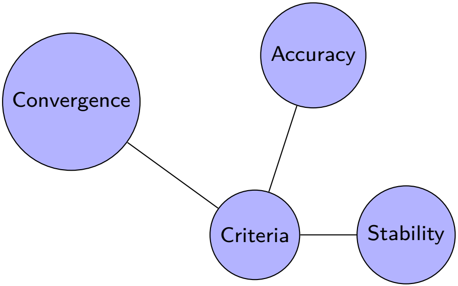

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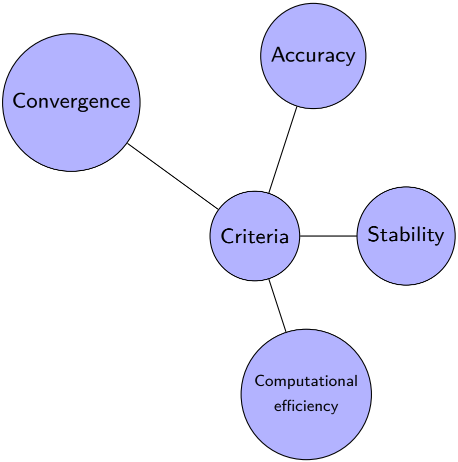

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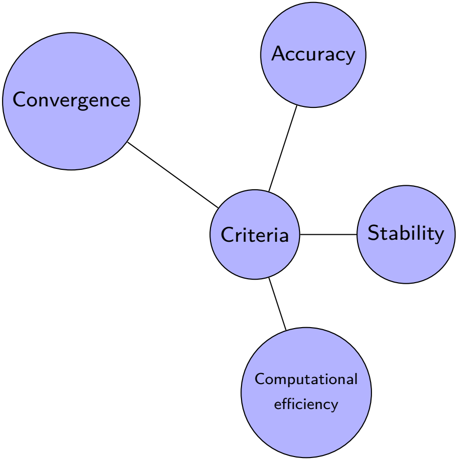

Possible source of instability and inaccuracies

Severe nonlinearity
Singularity
Ill-conditioned matrices
Topology changes
Time-step estimation
Numerical instability

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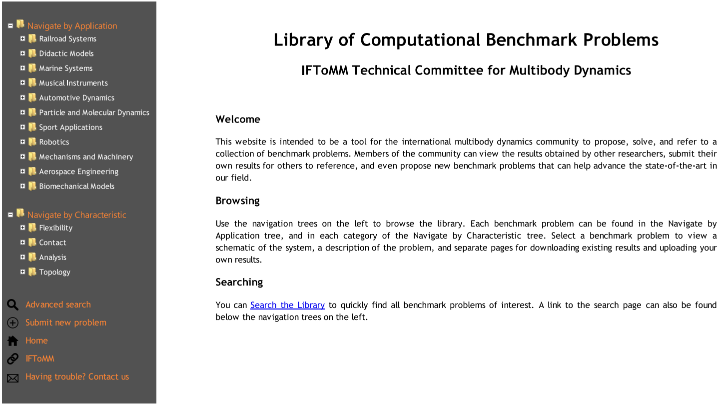
Online benchmark Library

A screenshot of the online Library:



Homepage Link

The online library

The Library provides users with the ability to:

- Describe new benchmark problems in a systematic way
- Locate all the information required to regenerate a multibody problem and technical data from the simulation results
- Communicate with other users to discuss issues and challenges regarding a particular multibody problem
- Compare and summarize different results from a benchmark problem
- Identify copyright agreements for submitted problems and novel procedures
- Share ideas for improving the benchmark library

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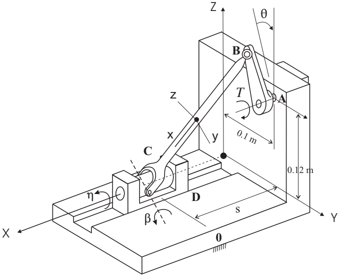
Uploading a benchmark problem

Key points in describing a new benchmark problem:

- Simplifications and assumptions
- Technical information: geometry, mass properties, effect of gravity, reference coordinate systems, and constant parameters
- Topology of the system, along with describing the operation of the mechanism
- Inputs, outputs, simulation time, initial conditions, and problem objective
- Numerical results, including plots and discussion
- Simulation issues: instabilities, singularities, and computational errors



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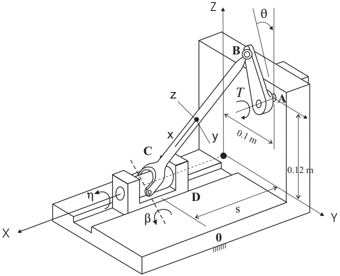

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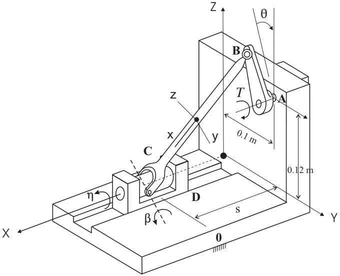

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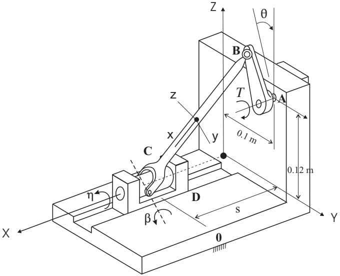

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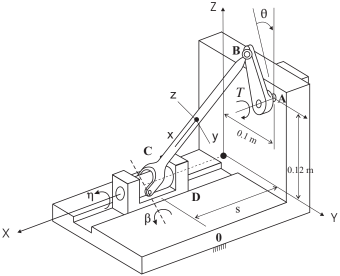

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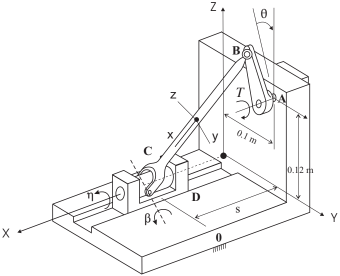
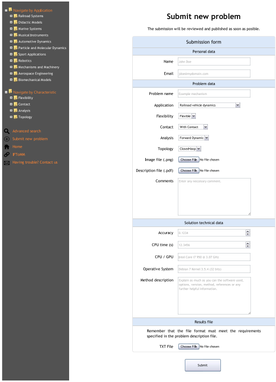


Simulation results submission

- **A screenshot from *technical details* page in the library:**



Details Link

Simulation results submission

✹ Problem description and simulation results submission

3D RIGID SLIDER-CRANK MECHANISM

Problem description

Figure 1 illustrates a spatial slider-crank mechanism[1], representing a four-body dynamic system. All links are rigid, subjected to gravity of magnitude 9.81 m/s^2 in the negative Z direction.

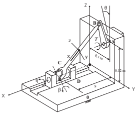


Figure 1: A spatial rigid slider-crank mechanism.

The mechanism consists of a crank AB of length 0.08 m, a connecting rod BC of length 0.3 m, and a sliding block. The crank, connected to the ground by revolute joint A, is driven from initial position $\theta = 0$ rad with initial angular speed of 6 rad/s. There is a spherical joint at B and a universal joint at C, with universal joint angles η and β defined in the figure. The block is constrained to the ground by a prismatic joint D with sliding displacement s.

Studying the forward dynamic response of the multibody system under the gravitational force, considering the above-mentioned initial conditions for the crank, is the main objective of this benchmark problem. Masses of the crank, connecting rod, and sliding block are respectively $m_{cr} = 0.12$ kg, $m_{co} = 0.5$ kg, and $m_s = 2.0$ kg. Mass moments of inertia for

[1] Edward J. Haug, *Computer Aided Kinematics and Dynamics of Mechanical Systems*. Vol. 1, Boston: Allyn and Bacon, 1989, pp.396–401.

1

the three bodies are:

$$I_{cr} = \begin{pmatrix} 0.0001 & 0 & 0 \\ 0 & 0.00001 & 0 \\ 0 & 0 & 0.0001 \end{pmatrix} \quad I_{co} = \begin{pmatrix} 0.004 & 0 & 0 \\ 0 & 0.0004 & 0 \\ 0 & 0 & 0.004 \end{pmatrix}$$

$$I_s = \begin{pmatrix} 0.0001 & 0 & 0 \\ 0 & 0.0001 & 0 \\ 0 & 0 & 0.0001 \end{pmatrix}$$

Joint-coordinate formulation is applied in the simulation model of the multibody system, considering the generalized coordinates as:

$$\mathbf{q} = \left\{\theta(t)\ \eta(t)\ \beta(t)\ s(t)\right\}^T$$

Four generalized coordinates coupled by three algebraic constraints leaves one degree of freedom for the mechanism, which can be simply perceived from the physics of the problem in this case. There is no friction in the joints.

Simulation results

MapleSim, a multi-domain modeling and simulation tool by Maplesoft, is used to simulate the 3D slider-crank mechanism. Graph-theoretic approach is used in MapleSim to introduce the topology of the system and formulate the system equations. A stiff DAEs solver is applied to acquire the simulation results.

Figure 2 shows the time history of the slider position and speed. Simulation results for

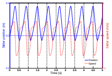


Figure 2: Time history of slider position and speed.

the crank angle are illustrated in Figure 3.

2

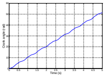


Figure 3: Time history of crank angle.

A text file from the simulation results data has been uploaded so that the user can regenerate the results for comparison purposes. The first column is time, the second one is the position data of the slider with respect to the global reference frame, and the third one is the crank angle data, starting from vertical position at which $\theta = 0$.

Table 1: Technical details for the problem, integrator, hardware, and software.

Integrator	Rosenbrock
Solver type	Stiff – variable time-step
Accuracy	xxx
Relative tolerance	1.10^{-4}
Absolute tolerance	1.10^{-4}
CPU time	250 ms
CPU/GPU	Intel(R) Core(TM2 Duo CPU E8400 3.00GHz 3.00 GHz)
Operating system	Windows7
Formulation procedure	Linear graph formulation
Dynamic equation	Symbolic, DAEs index-3
Programming language	MapleSim

3

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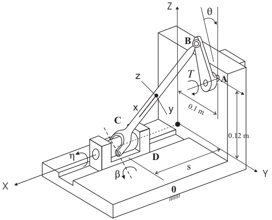

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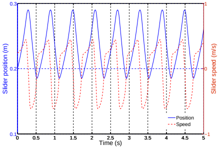

Figure 2: Time history of slider position and speed.

the crank angle are illustrated in Figure 3.

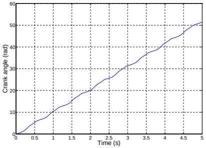

Figure 3: Time history of crank angle.

A text file from the simulation results data has been uploaded so that the user can regenerate the results for comparison purposes. The first column is time, the second one is the position data of the slider with respect to the global reference frame, and the third one is the crank angle data, starting from vertical position at which $\theta = 0$.

Table 1: Technical details for the problem, integrator, hardware, and software.

Integrator	Rosenbrock
Solver type	Stiff – variable time-step
Accuracy	xxx
Relative tolerance	1.10^{-4}
Absolute tolerance	1.10^{-4}
CPU time	250 ms
CPU/GPU	Intel(R) Core(TM2 Duo CPU E8400 3.00GHz 3.00 GHz)
Operating system	Windows7
Formulation procedure	Linear graph formulation
Dynamic equation	Symbolic, DAEs index-3
Programming language	MapleSim

Outline

1 Introduction

2 Systematic classification of multibody benchmark problems

3 Evaluation of benchmark problems

4 The Library of computational benchmark problems

5 **Summary and future work**

Summary & future work

Summary:

- An online Library of Computational Benchmark Problems has been designed.
- Some resources for well-established dynamic modeling, formulation, computer implementation, numerical integration, and computational procedures were presented.
- The general framework for the online Library was introduced.
- Some guidelines for using the Library were presented.
- A typical benchmark problem along with some screenshots from the Library were used to demonstrate the steps required to submit a benchmark problem to the Library.

Future work:

- The online Library has been designed to be flexible to changes in the classification scheme as deemed necessary, which will help ensure its longevity.
- There is ample room for improvement of the Library and evaluation principles by the multibody researchers community.

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It will be announced when the web page is ready for submissions!